Closing Thurs: 5.1/5.2, 5.3

Closing Sun: Optional Ch. 2 Review

Exam 2 is Tuesday, Nov. 22nd

No lecture Wednesday, Nov. 23rd.

5.3: More About Exponentials and Logs

Recall: Here is your full solving toolbox

Add./Sub.: $y = x + a \leftrightarrow \mathrm{y} - \mathrm{a} = x$

Mult./Div.: $y = mx \leftrightarrow \frac{\mathrm{y}}{\mathrm{m}} = x$

Powers/Roots: $y = x^n \leftrightarrow \mathrm{y}^{(1/n)} = x$

Exponentials/Logarithms:

$$y = b^x \leftrightarrow \log_{\mathrm{b}}(y) = x$$

"log base *b*"

Note: $\ln(y) = \log_{\mathrm{e}}(y)$

"natural logarithm"

Entry Task: Review of all solving. Get out your calculator and solve these equations

Equation	Inverse
$x + 3 = 14$ $y - 5 = 22$	$x =$ $y =$
$3t = 16$ $\frac{m}{0.2} = 100$	$t =$ $m =$
$x^2 = 7$ $\sqrt{y} = 3$	$x =$ $y =$
$t^5 = 20$ $\sqrt[5]{w} = 3$	$t =$ $w =$
$e^x = 10$ $\ln(y) = 3$	$x =$ $y =$
$5^t = 60$	$t =$

Another perspective: Many students think of inverses as "canceling".

Examples:

1. $x - 5 = 22 \rightarrow x - 5 + 5 = 22 + 5$

2. $3t = 16 \rightarrow \frac{3t}{3} = \frac{16}{3}$

3. $t^5 = 20 \rightarrow (t^5)^{\frac{1}{5}} = (20)^{\frac{1}{5}}$

4. $e^x = 10 \rightarrow \ln(e^x) = \ln(10)$

5. $\ln(y) = 3 \rightarrow e^{\ln(y)} = e^3$

All solving is just using inverses in the correct order.

Directly from Homework:

5.1/5.2: Problem 8

Solve $\frac{1}{6}e^{-4t} = 0.9$

5.1/5.2: Problem 10

Solve $8e^{2t-5} = 24$

5.3: Problem 5

Solve $65 = \frac{75}{1+5e^{-0.3x}}$

5.3: Problem 4

Solve $77 = 100 - 100e^{-0.08x}$

Basic Logarithm Facts

Rule
$1 = e^0$ so $\ln(1) = 0$ $e = e^1$ so $\ln(e) = 1$
$\ln(ab) = \ln(a) + \ln(b)$
$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
$\boldsymbol{\ln(a^b) = b\ln(a)}$
$\ln(e^x) = x$ $e^{\ln(y)} = y$

5.3 Problem 1:

Solve $6^{5x} = 56562$

5.3: Problem 2

Solve for x: $0.23P = P\,(2)^{-x}$

Aside (You don't need to write this)
Example to illustrate the rules

From last time, you should already know:

$$\frac{e^3 e^4}{e^2} = \frac{e^7}{e^2} = e^5$$

So

$$\ln\left(\frac{e^3 e^4}{e^2}\right) = \ln(e^5) = 5$$

But the rules say this is the same as

$$\begin{aligned}\ln\left(\frac{e^3 e^4}{e^2}\right) &= \ln(e^3 e^4) - \ln(e^2) \\ &= \ln(e^3) + \ln(e^4) - \ln(e^2) \\ &= 3 + 4 - 2 = 5\end{aligned}$$

So, in this says, the log rules are just a repeat of the exponent rules.

The Ch. 5 homework just gives practice with plugging in and solving. Here are the "hard" problems:

5.3: Problem 7: Given $P = Ae^{-0.05t}$
(a) A = 75000, P = 40000, what is *t*?
(b) P = 60000, t = 10, what is A?

5.3: Problem 8: Given $y = P_0 e^{ht}$
Given in 1998 (t=0), y = 100,000:
so $100000 = P_0 e^{h(0)}$
Given in 2008 (t=10), y = 110,365:
so $110365 = P_0 e^{h(10)}$
Asked: in 2023 (t=25), what is y?

5.3: Problem 9: Given $p = 100e^{-\frac{q}{2}}$
(a) If q = 4, what is p?
(b) If p = 2.01, what is q?

5.3: Problem 10:
Given $p = 500e^{-0.07x}$=price.
What is revenue when x = 100?

5.3: Problem 11: Given $S = 8500e^{0.039t}$
Note: 22 months is the same as $\frac{22}{12}$ *years*
(a) What is S when t = $\frac{22}{12}$ years?
(b) What is t when S is twice the initial value? (the initial value is 8500).

5.3: Problem 12: Given $S = 6000(1.005)^t$
(a) What is S when t = 1?
(b) What is t when S is twice the initial value? (the initial value is 6000).

5.3: Problem 13: Given $N = 500(0.1)^{0.7^t}$
(a) What is *N* when *t* = 0?
(b) What is *t* when *N* = 100?